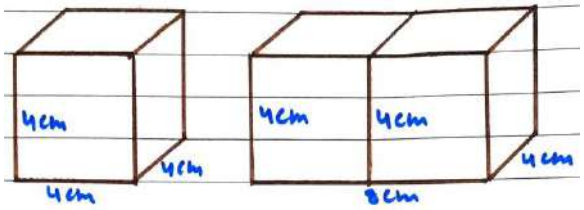
Exercise 12.1

Unless stated otherwise, take $\pi = \frac{22}{7}$

1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.



Sol. Volume of each cube (a^3) = 64 cm^3

$\Rightarrow a^3 = (4cm)^3$

$\Rightarrow a = 4$ cm

$\therefore$ Side of the cube = 4 cm

Length of the resulting cuboid = 4 cm + 4cm

= 8 cm

Breadth of the resulting cuboid = 4 cm

Height of the resulting cuboid = 4 cm

$\therefore$ Surface area of the cuboid = 2(lb + bh + hl)

$= 2(8\times4 + 4\times4 + 4\times8)\ cm^2$

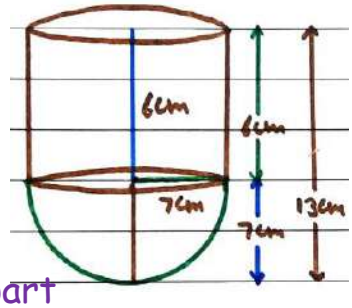
$= 2(32 + 16 + 32)\ cm^2$

$= (2 \times 80)\ cm^2$

$= 160\ cm^2$

$\therefore$ The surface area of the resulting cuboid = 160 cm^2 Ans.

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.



Sol. Diameter of the hemisphere = 14 cm

Radius of the hemisphere(r) = 7 cm

Height of the cylinder(h) = 13 - 7 = 6 cm

Also, radius of the hollow hemisphere = 7 cm

Inner surface area of the vessel

= CSA of the cylindrical part + CSA of hemispherical part

$= (2\pi rh + 2\pi r^2)\ cm^2$

$= 2\pi r\ (h + r)\ cm^2$

$= 2 \times \frac{22}{7} \times 7\ (6 + 7)\ cm^2$

$= 2 \times 22 \times 13\ cm^2$

$= 572\ cm^2$

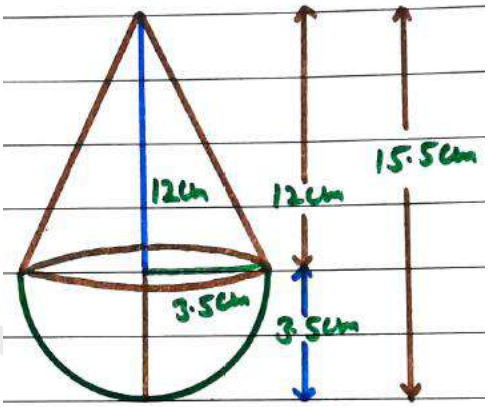
$\therefore$ The inner surface area of the vessel is 572 cm^2 Ans.

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Sol. Radius of the cone and the hemisphere(r) = 3.5 = $\frac{7}{2}$ cm

Total height of the toy = 15.5 cm

Height of the cone(h) = 15.5 cm - 3.5 cm
= 12 cm



$l^2 = h^2 + r^2$

$= (12)^2 + (\frac{7}{2})^2$

$= 144 + \frac{49}{4}$

$= \frac{576 + 49}{4}$

$= \frac{625}{4}$

$\therefore l = \frac{25}{2}$

Total surface area of the toy = CSA of cone + CSA of hemisphere

$= \pi rl + 2\pi r^2$

$= \pi r (l + 2r)$

$= \frac{22}{7} \times \frac{7}{2} \times (\frac{25}{2} + 2 \times \frac{7}{2})$ cm^2

$= 11 \times (\frac{25}{2} + 7)$ cm^2

$= 11 \times (\frac{25+14}{2})$ cm^2

$= 11 \times \frac{39}{2}$ cm^2

$= 11 \times 19.5$ cm^2

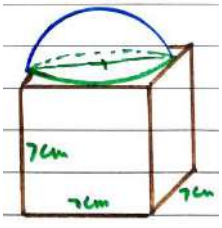
$\therefore$ The total surface area of the toy is 214.5 cm^2 Ans.

4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Sol. Each side of cube = 7 cm

$\therefore$ Greatest diameter the hemisphere can be 7cm

$\therefore$ Radius of the hemisphere = $\frac{7}{2}$ cm



Total surface area of solid

= S. A. of cubical block - Area of base of hemisphere + CSA of hemisphere

$= 6 \times s^2 - \pi r^2 + 2\pi r^2$

$= 6 \times s^2 + \pi r^2$

$= [6 \times 7^2] + [\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}]$

$= (6 \times 49) + \frac{77}{2}$

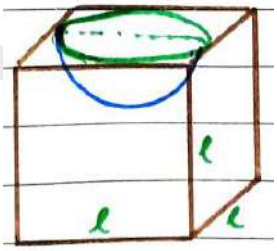
$= 294 + 38.5$

$= 332.5 \text{ cm}^2$

$\therefore$ The surface area of the solid is 332.5 cm^2 Ans.

5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Sol. Diameter of hemisphere = Edge of cube = l



$\therefore$ Radius of hemisphere = $\frac{l}{2}$

Total surface area of solid

= S. A. of cubical block –

Area of base of hemisphere + CSA of hemisphere

$= 6 \times l^2 - \pi r^2 + 2\pi r^2$

$= 6 \times l^2 + \pi r^2$

$= 6 \times l^2 + \pi \left(\frac{l}{2}\right)^2$

$= 6 \times l^2 + \frac{\pi l^2}{4}$

$= l^2 \left(6 + \frac{\pi}{4}\right)$

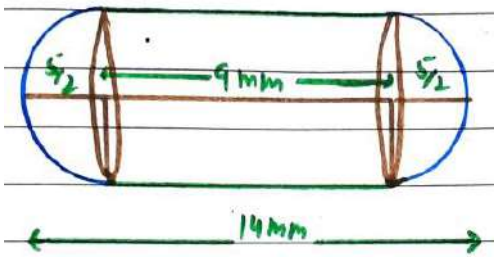
$= l^2 \left(\frac{24 + \pi}{4}\right)$ sq units

$\therefore$ The surface area of the remaining solid is $\frac{1}{4} l^2 (\pi + 24)$ sq units. Ans.

6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

Sol. Diameter of the capsule = 5 mm

$\therefore$ Radius = $\frac{5}{2}$ = 2.5 mm



Length of the entire capsule = 14 mm

$\therefore$ Length of the cylindrical part (h)

= 14 - (2.5 + 2.5)

= 9mm

$\therefore$ Required surface area of medicine capsule

= Surface area of cylinder + 2 × Surface area of hemisphere

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi r\,(h + 2\,r)$

$= 2 \times \frac{22}{7} \times \frac{5}{2} \times (9 + 2 \times \frac{5}{2})$

$= \frac{110}{7} \times (9 + 5)$

$= \frac{110}{7} \times 14$

$= 220\ mm^2$

$\therefore$ Surface area of capsule is 220 mm^2. Ans.

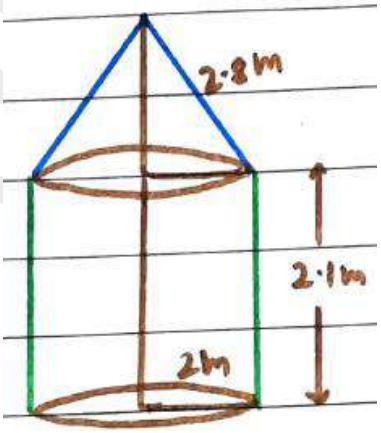
7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2.

Sol. Height the cylindrical part = 2.1 m.

Diameter of base of tent = 4 m

Radius of cylindrical and conical part = 2 m

Slant height of the conical part (l) = 2.8 m



$\therefore$ Surface area of tent

= S. A. of conical part + S. A. of cylindrical part

$= \pi rl + 2\pi rh$

$= \pi r\,(l + 2h)$

$= \frac{22}{7} \times 2\,(2.8 + 2 \times 2.1)$

$= \frac{44}{7}\,(2.8 + 4.2)$

$= \frac{44}{7} \times 7$

$= 44\ m^2$

The area of the canvas used for making the tent is 44 m^2

Cost of the canvas of the tent at the rate of ₹500 per m^2

= Surface area × cost per m^2

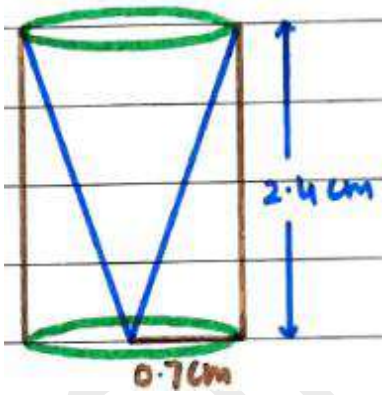
= 44 × 500

= ₹22000

$\therefore$ Cost of the canvas of the tent is ₹22000 Ans.

8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

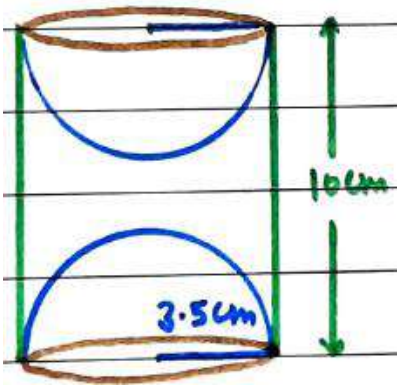
Sol. Height of cylinder = 2.4 cm
And diameter of cylinder = 1.4 cm
∴ Radius of cylinder = 0.7 cm
⇒ Height of conical cavity = 2.4 cm



And radius of conical cavity = 0.7 cm
For slant height of cone, $l^2 = h^2 + r^2$
$= (2.4)^2 + (0.7)^2$
$= 5.76 + 0.49$
$= 6.25$

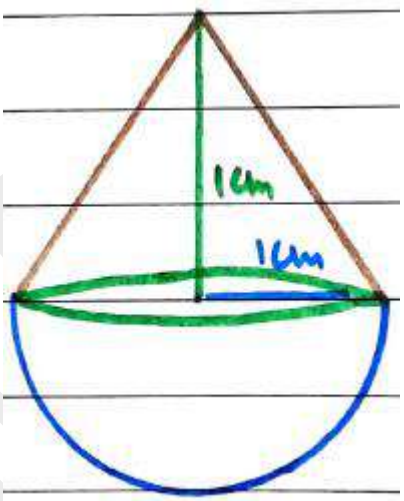
∴ $l = 2.5$ cm
TSA of remaining solid
= CSA of conical cavity + CSA of cylinder + Area of base
$= \pi rl + 2\pi rh + \pi r^2$
$= \pi r (l + 2h + r)$
$= \frac{22}{7} \times \frac{7}{10} \times (2.5 + 4.8 + 0.7)$
$= 2.2 \times 8$
$= 17.6 \text{ cm}^2$
$= 18 \text{ cm}^2$ (nearest)
∴ The total surface area of the remaining solid is 18 cm^2. Ans.

9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.
Sol. Height of the cylinder (h) = 10cm
and radius of base of cylinder (r) = 3.5 cm or $\frac{7}{2}$ cm
Radius of hemisphere (r) = 3.5 cm
Required total surface area of the article
= LSA of cylinder + 2 × CSA of hemisphere
$= 2\pi rh + 2 \times 2\pi r^2$
$= 2 \pi r (h + 2 r)$
$= 2 \times \frac{22}{7} \times \frac{7}{2} \times (10 + 2 \times \frac{7}{2})$
$= 22 \times (10 + 7)$
$= 22 \times 17$
$= 374 \text{ cm}^2$
∴ The total surface area of the article is 374 cm^2 Ans.



Exercise 12.2

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.



Sol. Radius of the base of cone (r) = 1 cm
Radius of the hemisphere (r) = 1 cm
and height of the cone (h) = 1 cm
∴ Required volume of the solid
= Volume of the cone + Volume of the hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2(h + 2\,r)$

$= \frac{1}{3}\pi \times 1 \times 1 \times (1 + 2 \times 1)$

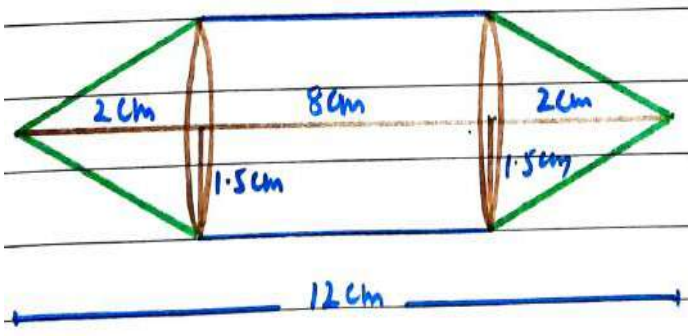
$= \frac{1}{3}\pi \times (1 + 2)$

$= \frac{1}{3}\pi \times 3$

$= \pi\ cm^3$

∴ The volume of the solid is $\pi\ cm^3$ Ans.

2. Rachel, an engineering, student was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made.



Sol. Length of model = 12 cm.
Height of conical part = 2cm
Height of cylindrical part
= 12cm – 2 cm – 2 cm
= 8 cm
Diameter of model = 3cm
Radius of cone and cylinder = $\frac{3}{2}$ cm
Volume of model = Volume of cylinder + 2 × Volume of a cone

$= \pi r^2 H + 2 \times \frac{1}{3}\pi r^2 h$

$= \pi r^2 (H + 2\,\frac{1}{3}\,h)$

$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}(8 + 2 \times \frac{1}{3} \times 2)$

$= \frac{99}{14} \times (8 + \frac{4}{3})$

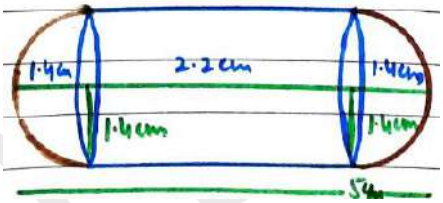
$= \frac{99}{14} \times \frac{28}{3}$

$= 33 \times 2 \text{ cm}^3$

$= 66 \text{ cm}^3$

$\therefore$ The volume of air contained in the model is 66 cm^3 Ans.

3. A Gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 Gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.



Sol. Length of Gulab Jamun = 5cm

Diameter of the Jamun = 2.8 cm

Radius of cylindrical and spherical part = 1.4 cm

$\therefore$ Height of cylindrical part = 5cm – 1.4 cm – 1.4 cm

= 2.2 cm

Volume of a Gulab Jamun

= Volume of cylinder + 2 volume of hemi-sphere

$= \pi r^2 h + 2 \times \frac{2}{3} \pi r^3$

$= \pi r^2 (h + \frac{4}{3} r)$

$= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10} (\frac{22}{10} + \frac{4}{3} \times \frac{14}{10})$

$= 22 \times \frac{2}{10} \times \frac{14}{10} \times (\frac{22}{10} + \frac{56}{30})$

$= \frac{616}{100} \times (\frac{66 + 56}{30})$

$= \frac{616}{100} \times \frac{122}{30} \text{ cm}^3$

Volume of 45 Gulab Jamuns = $45 \times \frac{616}{100} \times \frac{122}{30} \text{ cm}^3$

Volume of syrup = 30% of 45 Gulab Jamuns

$= 30\% \text{ of } 45 \times \frac{616}{100} \times \frac{122}{30} \text{ cm}^3$

$= \frac{30}{100} \times 45 \times \frac{616}{100} \times \frac{122}{30} \text{ cm}^3$

$= \frac{45 \times 616 \times 122}{10000} \text{ cm}^3$

$= \frac{3381840}{10000} \text{ cm}^3$

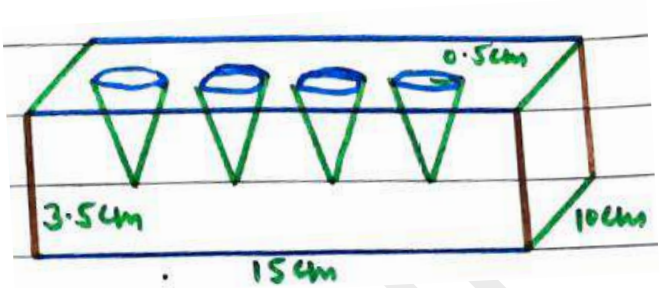
$= 338.184 \text{ cm}^3$

$= 338 \text{ cm}^3$ (approximate)

$\therefore$ Volume of syrup in 45 in Gulab Jamuns is 338 cm^3 Ans.

4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.

Sol. Length of cuboid (l) = 15 cm



Breadth of cuboid (b) = 10cm

and height of cuboid (h) = 3.5 cm

Radius of each of the depressions (r) = 0.5 cm or $\frac{1}{2}$ cm

and the depth (r) = 1.4 cm or $\frac{7}{5}$

∴ Volume of wood in the entire stand

= Volume of cuboid – 4 × volumes of a cone

= $l \times b \times h - 4 \times \frac{1}{3}\pi r^2 h$

= $15 \times 10 \times \frac{35}{10} - 4 \times \frac{1}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{7}{5}$

= $525 - \frac{22}{15}$

= 525 – 1.47

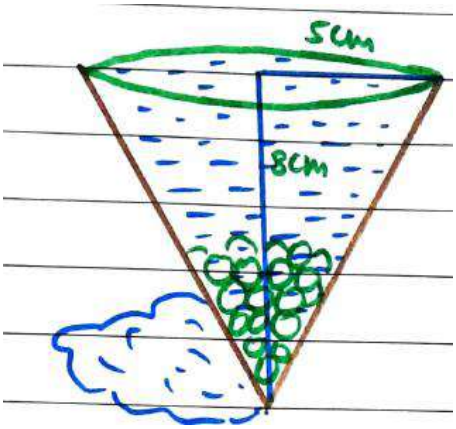
= 523.53 cm^3

∴ Volume of wood in the entire stand is 523.53 cm^3 Ans.

5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Sol. Height of inverted cone (h) = 8 cm

Radius of vessel (R) = 5 cm



Radius of sphere (r)= 0.5 cm

Numbers of lead

$$= \frac{\frac{1}{4}\text{volume of water flows out (volume of cone)}}{\text{Volume of a lead}}$$

$$= \frac{\frac{1}{4} \times \frac{1}{3}\pi R^2 h}{\frac{4}{3}\pi r^3}$$

$$= \frac{\frac{1}{4} \times \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 8}{\frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}}$$

$= 5 \times 5 \times 2 \times 2$

$= 100$

∴ The number of lead shots dropped in the vessel are 100 Ans.

6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use $\pi = 3.14$)

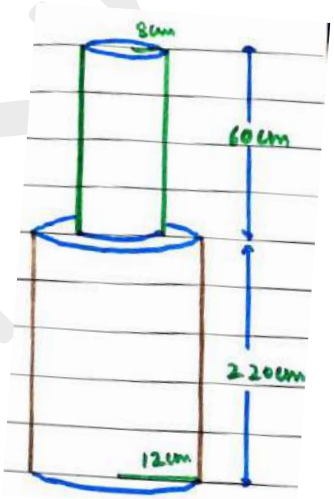
Sol. Height of pole (H)= 220 cm

Diameter of base of cylindrical pole = 24 cm

Radius of base of cylindrical pole (R)= 12 cm

Height of surmounted pole (h)= 60 cm

Radius of surmounted pole (r)= 8 cm



∴ Volume of pole

= Volume of cylinder + Volume of surmounted cylinder

$= \pi R^2H + \pi r^2h$

$= \pi (R^2H + r^2h)$

$= \frac{314}{100} (12 \times 12 \times 220 + 8 \times 8 \times 60)\ cm^3$

$= \frac{314}{100} (31680 + 3840)\ cm^3$

$= \frac{314}{100} \times 35520\ cm^3$

∴ The mass of the pole= $\frac{314}{100} \times 35520 \times 8$ gm

$= \frac{8922624}{10 \times 1000}$ kg

= 892.26 kg

∴ The mass of the pole is 892.26 kg. Ans.

7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Sol. Height of cone (h) = 120 cm

Radius of cone(r) = 60 cm

Radius of hemisphere (r) = 60 cm

Radius of cylinder (r) = 60 cm

Height of cylinder (H) = 180 cm

Volume of water left in cylinder

= Volume of cylinder – (volume of cone + volume of hemisphere)

$= \pi r^2 H - \left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)$

$= \pi r^2 \left(H - \frac{1}{3}h - \frac{2}{3}r\right)$

$= \frac{22}{7} \times 60 \times 60 \times \left[180 - \frac{1}{3} \times 120 + \frac{2}{3} \times 60\right]$

$= \frac{22}{7} \times 3600 \times [180 - 40 - 40]$

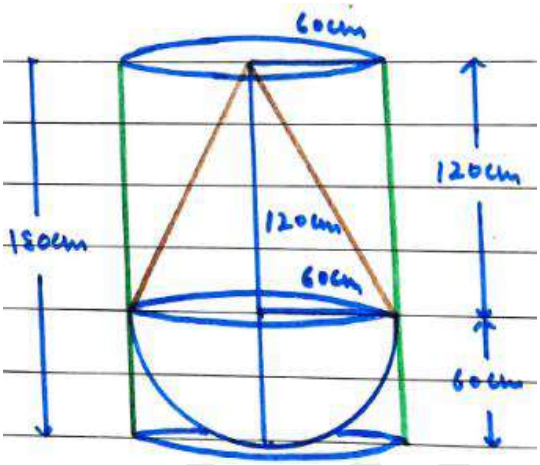
$= \frac{22}{7} \times 3600 \times 100$

$= \frac{7920000}{7}$

$= 1131428.5 \text{ cm}^3$

$= 1.131 \text{ m}^3$ (approximate)

∴ Volume of water left in cylinder is 1.131 m^3 Ans.



8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water, it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements and π = 3.14.

Sol. Height of neck (h)= 8 cm

Diameter of neck = 2 cm

∴ Radius of neck (r) = 1 cm

Diameter of sphere =8.5 cm

∴ Radius of sphere (R) = 4.25 cm

Volume of Vessel

= Volume of cylindrical neck + Volume of sphere

$= \pi r^2 h + \frac{4}{3}\pi R^3$

$= \pi \left(r^2 h + \frac{4}{3}R^3\right)$

$= 3.14 \times \left(1 \times 1 \times 8 + \frac{4}{3} \times \frac{17}{4} \times \frac{17}{4} \times \frac{17}{4}\right)$

$= \frac{314}{100} \times \left(8 + \frac{4913}{48}\right)$

$= \frac{314}{100} \times \left(\frac{384 + 4913}{48}\right)$

$= \frac{1663258}{4800}$

$= 346.51 \text{ cm}^3$

∴ She is not correct. Correct volume is 346.51 cm^3 Ans.